Mode Coupling and Leakage Effects in Finite-Size Printed Interconnects

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***Abstract*—A multimode analysis is used to describe how leakage effects are manifested in general printed interconnects situated on substrates of finite size. In the vicinity of discrete frequencies, it is shown that the analysis reduces to classical coupled-mode theory. The general results are then specialized to the particular case of shielded microstrip on an anisotropic substrate, for which numerical and experimental mode-coupling results are presented. The numerical results are demonstrated in the form of dispersion curves and field plots, and are computed using the finite-element method and the spectral-domain technique. The experimental results are performed using a network analyzer, and are given in terms of scattering parameters.**

***Index Terms*—Interconnect, leakage, mode coupling, transmission line.**

I. Introduction

FOR MANY years, printed transmission lines such as microstrip, slotline, coplanar strips, and coplanar waveguide were assumed to support modes with fields bound to the vicinity of the guiding region (slot or strip). More recently, however, it has been recognized that for certain geometrical parameters and frequencies, these structures can support modes which are leaky. This leakage occurs when the transmission-line mode propagates with a phase velocity faster than that of a mode or wave in the surrounding environment. Depending on the geometry and frequency of operation, leakage can occur in the form of a space wave [1], [2], surface wave [3]–[5], or a parallel-plate mode [6], [7] (for a structure open above and/or below, leakage can occur simultaneously in the form of a space wave and a surface wave [2]). In this paper, we are concerned with leakage effects involving transmission lines in practical integrated circuits. Such transmission lines are usually designed so as to only propagate their fundamental (zero-cutoff frequency) mode(s) in the frequency regime of interest and, therefore, the emphasis here is on potential leakage from such modes.

It is well known that the fundamental mode of a realistic printed transmission line has, for all frequencies, a phase velocity which is slower than the speed of light in free space. Therefore, such a mode can only leak energy in the form of a surface wave; for circuits with top and bottom conducting plates, the surface-wave mode is more accurately termed a parallel-plate mode. However, in most cases, the parallel-plate fields into which leakage occurs are essentially the same as surface-wave fields and, therefore, under such circumstances the physical mechanisms involved in leakage to a surface wave and a parallel-plate mode are very similar. Therefore, throughout the remainder of this paper, the leakage is referred to as being in the form of a surface wave, with the understanding that in some cases it is more rigorously described as a parallel-plate mode. Leaky-wave modes, characterized by complex propagation constants, can only exist in an idealized structure with an infinite region into which the energy can leak. For the case of a printed transmission-line, a leaky-wave mode only exists if the substrate is of infinite transverse extent. Several researchers have investigated potential leakage from printed transmission lines on idealized substrates of infinite size. For example, leakage has been investigated from coplanar strips [1], [5], coplanar waveguide [3], slotline [3], [6], microstrip on an anisotropic substrate [4], and from broadside-coupled microstrip [7]. Experiments have also been performed on slotline [3]; in these experiments, a microwave absorber was used on the sides of the structure to simulate a geometry of infinite transverse extent.

An actual integrated circuit is obviously printed on a substrate of finite size. Although the leaky-wave mode analyses and experiments discussed above have been useful in elucidating the dominant physical effects associated with leakage in printed transmission lines and in identifying various structures that can be leaky, they have not dealt with how leakage is manifested in a practical circuit of finite size. We present here a general theory that describes how a leaky wave from a printed transmission line on a substrate of infinite width is manifested on a substrate of finite width. The analysis uses multimode theory, and is applicable to any finite-size printed transmission line. At and around particular discrete frequencies, the multimode analysis reduces to classical coupled-mode theory. Subtle distinctions are made between the perturbed modes from idealized structures and the actual modes that exist on the transmission line of finite size. The ideas gained from the multimode analysis are then demonstrated for the particular case of leakage from the fundamental mode of microstrip on an anisotropic substrate of finite width. Both numerical and experimental results are presented for this structure. The numerical results are computed using the finite-element method and the spectral-domain technique, and are presented in the form of dispersion curves, field plots, and power density plots, which clearly show the characteristics of classical coupled-mode theory. Additionally, we examine this structure experimentally, with results presented in the form of scattering parameters.

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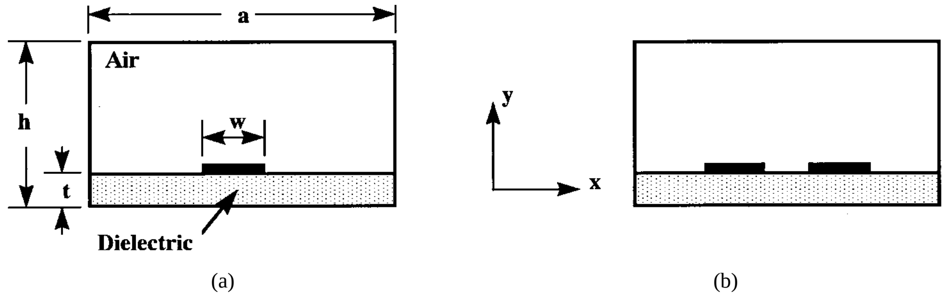

Fig. 1. (a) Cross section of shielded microstrip. (b) Conductor-backed coplanar strip waveguide.

II. Multimode Description

A. Physical Concept

Consider a leaky fundamental mode associated with an arbitrary printed transmission line. To speak of a leaky mode, it is implied that the substrate on which the line is printed is of infinite transverse extent. Assume also that this mode is excited by a time-harmonic source at the point $z = 0$, and the transmission line extends along the z-axis from $z = 0$ to infinity. If the frequency of the source is such that the fundamental transmission-line mode is leaky, the amplitude of the field in the vicinity of the slot or strip (slot for slotline-like structures, and strip for microstrip-like structures) decreases with distance along the longitudinal direction, as the energy leaks into the substrate. The above scenario, for a printed transmission line on a substrate of infinite extent, is referred to as Case 1.

Now, consider the same transmission line, but with its transverse extent truncated in some way; we refer to this as Case 2. This truncation may be manifested, for example, in slotline or coplanar waveguide with finite-width conducting plates, or microstrip or coplanar strips on substrates of finite width (see Fig. 1). It is important to stipulate that the truncation must be sufficiently far away from the guiding slot or strip such that the fields at the launch are the same as those in the idealized infinite structure in Case 1. If we excite Case 2 in the same manner as Case 1, we would expect the energy to again initially leak at an angle (in a cone) from the guiding region (unaware, initially, of the lateral truncation). Until the leaked energy encounters the sidewalls, the field profiles in Cases 1 and 2 are identical. After some longitudinal distance (z_o), the leaked energy in Case 2 encounters the sidewalls, and reflects back toward the guiding region.

B. Multimode Analysis

For simplicity, in the remainder of this paper, Case 2 is specialized to an interconnect placed in the center of a rectangular waveguide (this is shown in Fig. 1 for the case of microstrip and conductor-backed coplanar strips). Case 1 is again the corresponding transmission line on an infinite substrate (no sidewalls). Since in Case 2 the transmission line is placed inside the waveguide, only modes with purely real or imaginary propagation constants are expected (neglecting the possible complex modes that may exist in complex conjugate pairs [8]). Such that the subsequent discussion is clear, it is important to carefully define the various modes that can be supported by the structure in Case 2. In the absence of the printed transmission line, the structure supports box modes. Thus, in Case 2 there is a class of modes predominantly associated with the box and with slight perturbation caused by the presence of the printed transmission line. These modes are termed "perturbed box modes." The second class of modes in Case 2 are associated with the printed transmission line in Case 1, and are perturbed by the presence of the box. These are called "perturbed transmission-line modes." The perturbation of the box modes is often weak since the strips usually occupy only a small part of the cross section. However, the perturbation of the transmission-line modes can be severe, since we can go from a nonspectral mode with complex propagation constant (leaky-wave mode) to a spectral mode with a real or imaginary propagation constant. The cutoff frequencies of the perturbed box modes are determined predominantly by the size of the box, while the perturbed transmission-line modes have cutoff frequencies that are determined predominantly by the width of the strip or slot.

We assume Case 1 is excited such that the dominant transmission-line mode is launched, and that this mode is leaky. In Case 2, we consider the same excitation. Assume further that the leaky-wave wavenumber in Case 1 is β, and the surface wave to which it leaks has wavenumber β_{SW} ($\beta < \beta_{SW}$). The angle of leakage is, therefore, $\theta \approx \cos^{-1}(\beta/\beta_{SW})$, and the distance z_o at which the leaked energy first hits the sidewall in Case 2 is approximately $z_o \approx a/(2\tan\theta)$, where the box has a width a. Depending on the leakage angle θ and the box width a, the distance z_o can be either large or small. In either situation, we can always express the fields in the cross section of Case 2 as an infinite summation of spectral modes (the propagating and nonpropagating perturbed box and transmission-line modes).

We first consider the case for which z_o is large. A "large" distance is defined as one at which the cutoff modes contribute negligibly to the total field. For such a case, the cutoff modes and *multiple* propagating modes describe the complicated field cone emanating from the launch; more than one propagating mode is required since no one propagating mode can represent the leakage cone from the launch up to the point z_o. The modes construct the field profile at $z = 0$, and because these modes propagate at different velocities, they get out of phase with increasing z and combine to represent the complicated field profile from $z = 0$ to z_o, and beyond. The other extreme is when z_o is very small (large leakage angle and/or narrow box), such that the complicated fields to z_o need not be described by more than one propagating mode. Such a situation could be obtained by diminishing the box width until all perturbed box modes are cutoff. In such a situation, the fields must be described by the fundamental perturbed transmission-line mode and an infinite sum of cutoff modes. In the first extreme, for all z there is more than one propagating mode; in the second extreme, after a sufficient distance from the launch, only the perturbed transmission-line mode is propagating. One should recall, however, that this discussion began by requiring that the sidewalls be sufficiently distant from the printed transmission line such that the fields at $z = 0$ in Cases 1 and 2 are the same. In the second extreme, we required the sidewalls to be so close to the transmission line such that z_o was very small. In such a scenario, it is likely that the sidewalls perturb the total fields at $z = 0$, violating our original assumption. In most circuits of practical interest, the box walls are distant from the printed transmission line such that z_o is large. For the remainder of the discussion, we restrict ourselves to this situation, with the understanding that it is possible to eliminate the propagating box modes by shrinking the distance between the sidewalls.

Although the above discussion is a straightforward application of modal theory, it has important implications. Often when shielded planar transmission lines are modeled, one calculates a single real propagation constant, and assumes the fields of the corresponding mode describe the fields of the guide for all z. The above discussion shows that when the transmission line would be leaky in a transversely open structure (Case 1), often more than one propagating mode is required to describe the field in a corresponding shielded structure (Case 2). This observation is consistent with a recent paper by Tsuji and Shigesawa [9], in which they showed that if a leaky planar transmission line is placed in a box, it displays a pulse distortion that is inconsistent with the dispersion of a single shielded transmission-line mode.

C. Physical Effects

In Case 2, since the transmission line is placed inside the waveguide, only modes with purely real or imaginary propagation constants are expected (neglecting again the presence of possible complex modes). At frequencies for which the *perturbed* transmission-line mode has a wavenumber β_t which is less than that of a surface wave β_{SW} supported by the substrate, the fields of the perturbed transmission-line mode, away from the strip(s), can be represented in terms of propagating surface waves traveling at an angle from the strip(s) (see Fig. 2). The wavenumber of the perturbed transmission-line mode β_t in Case 2 should be contrasted with the unperturbed leaky-wave mode in Case 1, which has a complex propagation constant $\alpha + j\beta$, with $\beta_t = \beta + \epsilon$. Usually, the real number ϵ is small, such that the transmission-line mode in Case 1 becomes leaky ($\beta < \beta_{SW}$) at or near the frequency when $\beta_t < \beta_{SW}$. This implies that at frequencies for which the transmission-line mode in Case 1 is leaky, the perturbed transmission-line mode in Case 2 consists of fields away from the strip that are characterized by surface waves traveling at an angle. Moreover, the surface waves that compose the fields away from the slot or strip in the transmission-line mode of Case 2 are the same waves to which leakage occurs in Case 1.

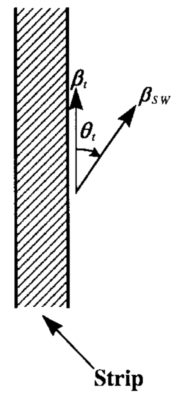

Fig. 2. Schematization of the fields of a perturbed transmission-line mode that is a fast wave with respect to a surface wave in the surrounding dielectric. When the perturbed transmission-line mode becomes a fast wave with respect to a surface wave in the surrounding dielectric ($\beta_t < \beta_{SW}$), the energy of the perturbed mode away from the strip is in the form of the surface wave with wavenumber β_{SW} traveling at an angle $\theta_t = \cos^{-1}(\beta_t/\beta_{SW})$ (there is also a surface wave traveling back to the strip due to reflection at the side terminations).

D. Classical Mode Coupling

The unperturbed box modes can be represented as a superposition of surface waves bouncing at an angle between the sidewalls. The longitudinal wavenumber $\beta_{\text{box}\,n}$ of the nth unperturbed box mode (n being a nonnegative integer) produced by the surface wave with wavenumber β_{SW} can be expressed as $\beta_{\text{box}\,n} = [\beta_{SW}^2 - (n\pi/a)^2]^{1/2}$ for a box of width a. There are, therefore, an infinite number of discrete box modes associated with each surface wave. We again assume the perturbation of the box modes due to the strip(s) is weak, and that the wavenumbers of the perturbed and unperturbed box modes are essentially equal. The angle at which the surface waves leave the perturbed transmission-line mode (under leakage-like conditions) is given as before by $\theta_t = \cos^{-1}(\beta_t/\beta_{SW})$, and the surface waves associated with the perturbed box modes travel at angles given by $\theta_{\text{box}\,n} = \cos^{-1}(\beta_{\text{box}\,n}/\beta_{SW})$. If $\beta_t = \beta_{\text{box}\,n}$, $\theta_t = \theta_{\text{box}\,n}$, and one would expect strong interaction between the perturbed transmission-line and box modes. This is the same condition as that for classical mode coupling [10] (see Fig. 3).

As the perturbed transmission-line mode first becomes a fast wave with respect to the surface wave in the surrounding dielectric (as a function of frequency), the surface waves excited in the surrounding substrate travel in the direction of the transmission line. With increasing frequency, the angle of propagation of the surface wave sweeps out away from the strip as β_t becomes smaller with respect to β_{SW} (see Fig. 2). The condition $\beta_t = \beta_{\text{box}\,n}$ can also be expressed as $\beta_{tx} = n\pi/a$ with $\beta_{tx} = [\beta_{SW} - \beta_t]^{1/2}$. This expression indicates that as the box width a increases, the frequency intervals between which classical mode coupling occurs becomes smaller. In the limit as the width goes to infinity, these mode-coupling regions occur continuously with increasing frequency. This, therefore,

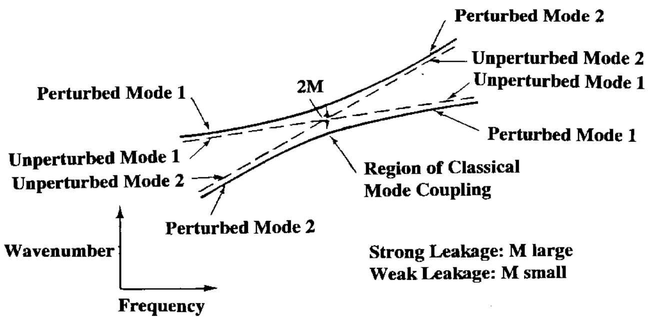

Fig. 3. Example dispersion curves characteristic of classical mode coupling. The dashed lines correspond to the two unperturbed modes from two distinct waveguides. In our case, one mode corresponds to a box mode in the absence of the printed transmission line and the other mode corresponds to the transmission line in the absence of the box. When the printed transmission line is placed inside the box, the two modes are perturbed. Instead of crossing, the dispersion curves of the perturbed modes (solid curves) bend away from each other. The coupling parameter M indicates the coupling strength. For strong coupling, M is large, with the opposite being true of weak coupling.

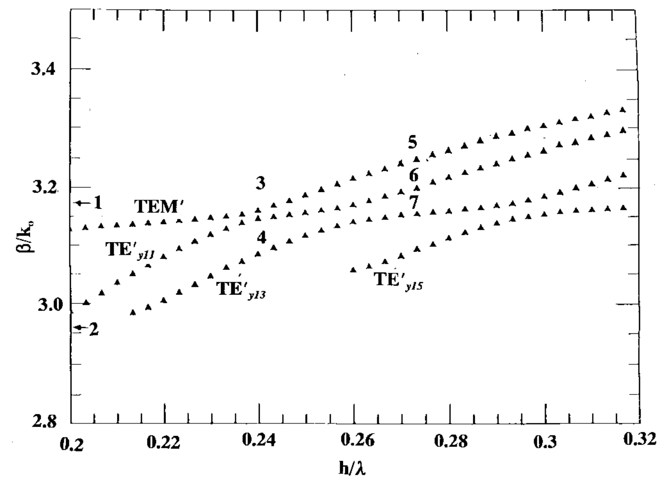

Fig. 4. Dispersion curves for shielded microstrip on an Epsilam-10 substrate ($\epsilon_{\|} = 10.3$ and $\epsilon_{\perp} = 13$). The optical axis is perpendicular to the strip surface. Referring to Fig. 1, the geometrical dimensions are $w = t$, $h = 5t$, and $a = 9t$. TEM' denotes the perturbed transmission-line mode and the TE'_{y1n} are perturbed box modes. The points identify points in the dispersion curves for which field plots are presented in Figs. 5 and 6.

is analogous to the case of an ideal leaky transmission line on a substrate of infinite cross section on which leakage occurs continuously in particular frequency intervals.

The relationship between transmission-line leakage in Case 1 and mode coupling in Case 2 can be made explicit. As discussed above, mode coupling between perturbed transmission-line modes and perturbed box modes occurs at frequencies where the dispersion curves of the unperturbed modes cross. If a particular transmission-line mode in Case 1 is nonleaky, then $\beta > \beta_{SW}$, where β is the wavenumber of the transmission-line mode and β_{SW} represents the wavenumber of *any* surface wave supported by the dielectric slab. Further, we recall that the wavenumber of the nth box mode associated with β_{SW} is $\beta_{\mathrm{box}\,n} = [\beta_{SW}^2 - (n\pi/a)^2]^{1/2}$. Thus, if a particular transmission-line mode in Case 1 is nonleaky for all frequencies, then $\beta > \beta_{\mathrm{box}\,n}$ for all frequencies (for all box modes); under this condition there can be no mode crossings of the unperturbed modes and, therefore, there can be no mode coupling between the perturbed modes in the shielded transmission line. This simple proof shows that mode coupling between a perturbed transmission-line mode and perturbed box modes can occur *only* if the unperturbed transmission-line mode, in the absence of the shield is leaky. This demonstrates that leakage, which heretofore has only been investigated for the case of open structures, has a direct effect on the physics of shielded transmission-line structures.

III. Example Results

Several issues have been addressed with respect to leakage-like effects associated with printed transmission lines on substrates of finite size. Below, we discuss theoretical and experimental results that quantify some of these points.

A. Numerical Calculations

We consider microstrip printed on Epsilam-10, a uniaxial anisotropic substrate. The optical axis is perpendicular to the surface of the strip, with $\epsilon_{\|} = 10.3$ and $\epsilon_{\perp} = 13.0$. The calculations have been performed using the spectral-domain technique [11] and the finite-element method [12]. First, consider an idealized case for which the microstrip is printed on such a substrate, and the substrate (and ground plane) has infinite transverse extent (as in Case 1 above). The dominant microstrip mode has its electric field lines predominantly along the optical axis and, therefore, will have a high-frequency effective dielectric constant near $\epsilon_{\|}$. It can be shown, however, that the TE_z surface waves supported by such a uniaxial anisotropic substrate have high-frequency effective dielectric constants approaching $\epsilon_{\perp}$. Thus, on a uniaxial anisotropic substrate such as Epsilam-10 (for which $\epsilon_{\perp} > \epsilon_{\|}$), at high frequencies the microstrip mode is a fast wave with respect to one or more TE_z surface waves, leading to leakage. It has been shown that a fundamental microstrip mode on such a uniaxial substrate is indeed leaky at high frequencies, with the energy leaked away from the strip in the form of a TE_z surface wave [4]. Here, we consider the same structure placed in rectangular waveguide (as in Case 2, see Fig. 1). The microstrip line is centered in the waveguide. The dimensions of the structure are normalized to the dielectric thickness t with $w = t$, $a = 9t$, and $h = 5t$ (see Fig. 1). A portion of the normalized dispersion curve is shown in Fig. 4 for the shielded microstrip structure. The mode designation TE_{ymn} corresponds to the mth root of the transcendental equation for the TE_y box mode with transverse wavenumber $k_x = n\pi/a$, for a box of width a. The modes in Fig. 4 are identified as TEM' and TE'_{ymn}, to denote the perturbed transmission-line mode and perturbed box modes, respectively. The geometrical parameters selected are identical to those used in [4] (with the addition of sidewalls and a top cover) for which a leaky transmission-line mode was supported. The leakage in [4] started at $t/\lambda \approx 0.24$, and Fig. 4 shows the dispersion curve in this region. For $t/\lambda < 0.2$, the dispersion curve of the perturbed transmission-line mode (TEM') exhibits very little dispersion and the dispersion curves of the perturbed box modes approach cutoff quickly $(\beta/k_o = 0)$ with decreasing t/λ.

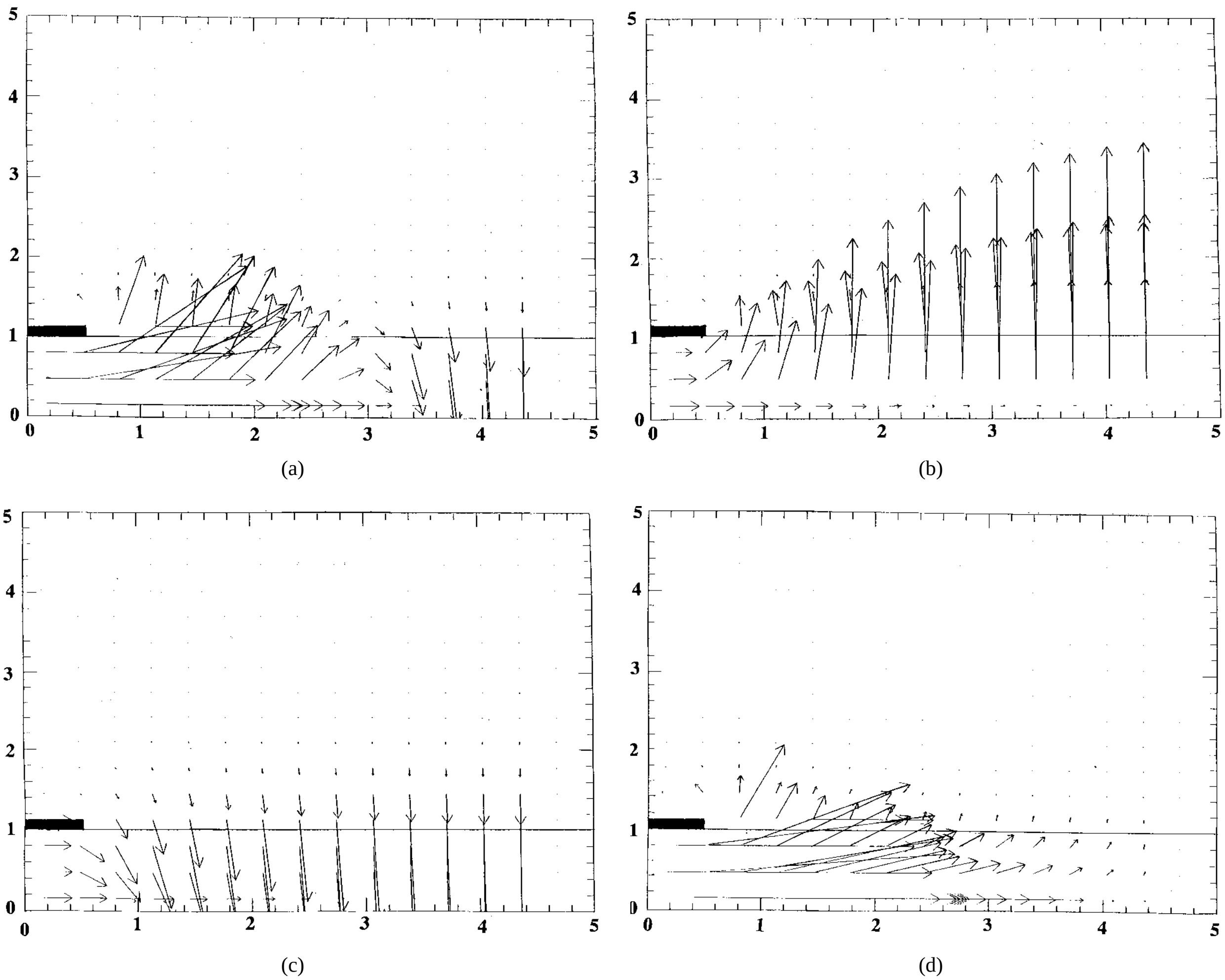

Fig. 5. Transverse magnetic field associated with the modes in Fig. 4 at Points 1–4. The plots are drawn as follows: the point from which the arrow *originates* corresponds to the point in space at which the field is calculated, the arrow indicates the direction of the field, and the length of the arrow represents the relative strength of the field at that point. Curves (a)–(d) correspond to Points 1–4 in Fig. 4, respectively. Fig. 5(a) and (b) was calculated at $t/\lambda = 0.21$ and (c) and (d) was calculated at $t/\lambda = 0.25$.

The TE_{y1n} box modes can be represented as TE_{x1} surface waves bouncing at an angle between the sidewalls, with transverse wavenumber $k_x = n\pi/a$. Further, as stated above, classical mode coupling is expected when $\beta_t \tan\theta_t \approx n\pi/a$, for a perturbed transmission-line mode of wavenumber β_t and a surface wave with wavenumber β_{SW}, where $\theta_t = \cos^{-1}(\beta_t/\beta_{SW})$. For the structure in Fig. 4, the box is relatively wide and, therefore, the first region of classical mode coupling occurs at frequencies just after the point at which the perturbed transmission-line mode becomes a fast wave with respect to the TE_{z1} surface wave. Additionally, other regions of classical mode coupling are evident with increasing frequency. In this example, mode coupling occurs only for perturbed box modes with odd n. This is explained by the even symmetry about the strip center of the y-directed electric fields for the fundamental microstrip mode. The perturbed box modes with odd n have y-directed electric fields with even symmetry about the center of the box, while modes with even n have odd symmetry. If the strip was offset between the two sidewalls, the perturbed transmission line would display classical mode coupling with perturbed box modes of both symmetries.

In addition to considering the dispersion curves, it is of interest to investigate the field profiles of the modes in Fig. 4. From Fig. 3, one observes that in a region of mode coupling there is a switch of mode types on either side of the coupling region. This is a well-known property of classical mode coupling [11]. In Fig. 5(a)–(d) are shown the transverse magnetic fields at Points 1–4, respectively, in Fig. 4. At Point 1 [Fig. 5 (a)], the magnetic fields are nearly confined to and encircle the strip, as expected of the fundamental (quasi-TEM) microstrip mode. By contrast, Point 2 [Fig. 5(b)] corresponds to a perturbed box mode, with fields unconfined to the strip region. At Point 3 [Fig. 5(c)], the field is again unconfined to the vicinity of the strip, and is in the form of a perturbed box mode. Finally, at Point 4 [Fig. 5(d)], the fields are predominantly confined to the vicinity of the strip, in the form of a perturbed transmission-line mode.

At Point 1 [Fig. 5(a)], the microstrip mode is still a slow wave with respect to the TE_{z1} surface wave and, therefore, the perturbed microstrip mode looks much like a quasi-TEM mode: the fields away from the strip are very small compared to those around the strip. However, at Point 4 [Fig. 5(d)], while the magnetic fields are primarily circling the strip, the relative

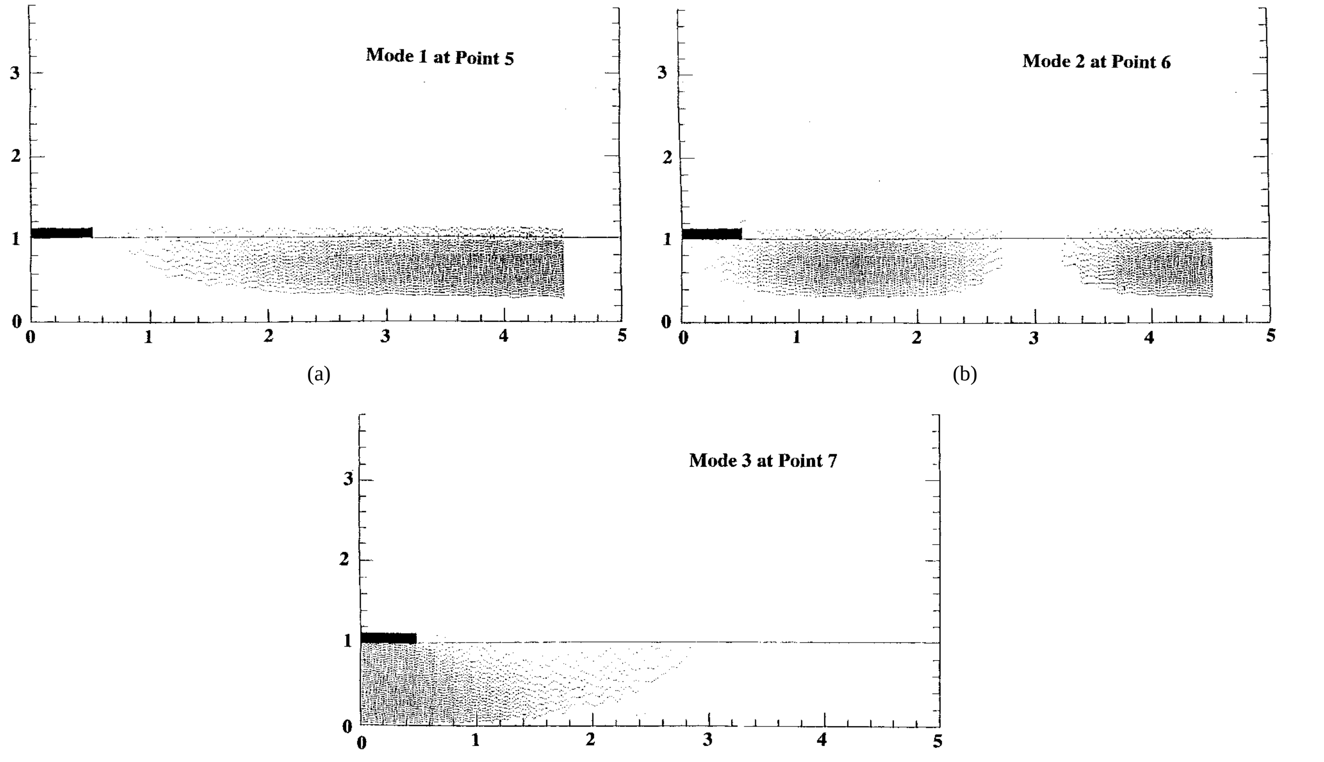

Fig. 6. Grayscale plots showing the cross-sectional power density for three of the modes in Fig. 4. All calculations were performed at $t/\lambda = 0.27$, and (a)–(c) correspond to Points 5–7 in Fig. 4. The regions from high-to-low relative power density are represented with regions of dark-to-light shading, respectively.

field strengths away from the strip are much larger than those in Fig. 5(a). This is because at Point 4 the transmission-line mode is a fast wave with respect to the TE_{z1} surface wave. Thus, although the fields look like those of a classical (quasi-TEM) microstrip mode in the strip region, the fields away from the strip are nonnegligible (and are in the form of a propagating surface wave).

A more dramatic demonstration of this phenomenon is shown in Fig. 6, in the form of power-density plots for Points 5–7 in Fig. 4. In these figures, it is clear that the fields at Points 5 and 6 correspond to perturbed box modes while the fields associated with Point 7 correspond to a perturbed transmission-line mode. This well-known mode switching associated with classical coupled-mode theory has important implications with respect to the modeling of the modes of such structures. Usually, when modeling printed transmission lines, it is assumed that the mode with the largest propagation constant corresponds to the dominant transmission-line mode. These power density plots clearly demonstrate that this is not the case in regions of classical mode coupling.

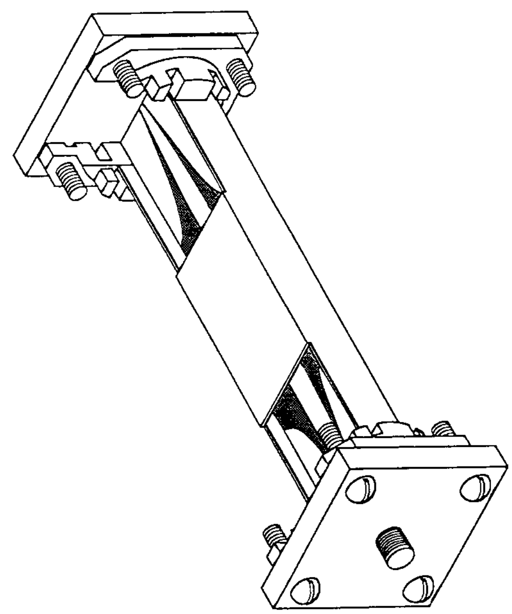

Fig. 7. Test set used to perform measurements.

B. Experimental Results

In addition to calculations, we have performed measurements to examine some of these effects in actual printed transmission lines on substrates of finite size. A schematic of the device on which the measurements were performed is shown in Fig. 7. We constructed two structures. One consisted of microstrip on an Epsilam-10 substrate with all dimensions exactly as in Figs. 4–6. The second structure was identical to the first, except instead of using Epsilam-10 as a substrate, we used Rogers Duroid ($\epsilon_r \approx 11$). This isotropic substrate was chosen because it has a dielectric constant which is close to both of the dielectric constants associated with the uniaxial anisotropic substrate Epsilam-10. The measurements were performed with an HP8510B network analyzer. With the bandwidth available, we were required to use thick substrates (5.08 mm) such that we could see effects associated with

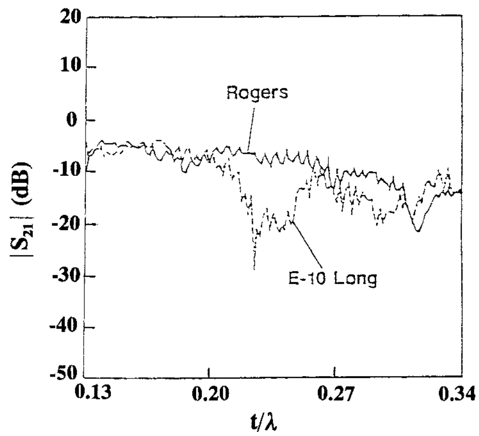

Fig. 8. Magnitude of $|S_{21}|$ for microstrip transmission line printed on the anisotropic substrate Epsilam-10 (E-10) and on Rogers Duroid, an isotropic substrate ($\epsilon_r \approx 11$). The geometrical parameters are identical to those used in Fig. 4. As indicated in Fig. 4, the microstrip mode on E-10 becomes a fast wave for $t/\lambda \approx 0.24$, and at higher frequencies, classical mode coupling occurs. Both microstrip lines are 5-cm long.

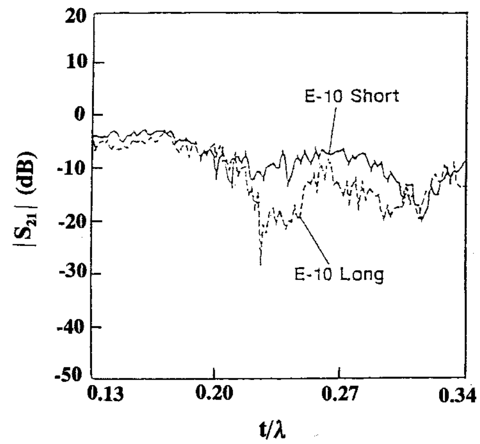

Fig. 9. Magnitude of S_{21} for microstrip line printed on Epsilam-10 (E-10), with all cross-sectional dimensions as in Fig. 4. One curve (E-10 Short) corresponds to microstrip of a few millimeters in length, while the other (E-10 Long) corresponds to a 5-cm-long microstrip line.

Figs. 4–6. With such a thick substrate, we were concerned that it would be difficult to excite the microstrip mode directly. Therefore, we used a coplanar-waveguide-to-microstrip transition (Fig. 7) to effectively excite the microstrip mode. Measurements are shown in Fig. 8 for the transmitted signal $|S_{21}|$ for the microstrip on the Epsilam-10 and Rogers substrates. The length of the microstrip line in both cases is 5 cm. Over most of the measured bandwidth, the S_{21} associated with the Rogers' substrate is relatively small and frequency independent. However, for the microstrip on Epsilam-10, a sharp dip in S_{21} is observed very near $t/\lambda = 0.24$. This was the point at which the first coupling behavior is expected (Fig. 4). At frequencies above which the perturbed transmission-line mode is a fast wave, there is a strong drop in $|S_{21}|$ associated with the Epsilam-10 substrate not seen for Rogers Duroid. We did a second set of measurements to investigate the effect of the transmission-line length on this phenomenon. We constructed a geometry for which the microstrip was again fabricated on Epsilam-10, but it was only a few millimeters long. In Fig. 9, results are shown which compare $|S_{21}|$ for the long (5 cm) and short (few millimeters) microstrip line on Epsilam-10. The change in $|S_{21}|$ is far less dramatic for the short transmission line when compared to the longer line.

We can interpret these results using several of the concepts developed above. At frequencies for which the microstrip mode is a slow wave with respect to the surface waves supported by the dielectric, the fields associated with the microstrip mode are bound to the vicinity of the strip. Thus, at frequencies for which the microstrip mode on Epsilam-10 is such a slow wave, it will behave much like the microstrip mode associated with Roger's Duroid (which is always a slow wave). However, at frequencies for which the microstrip mode associated with the Epsilam-10 substrate is a fast wave with respect to a surface wave, the fields "leak" away from the strip. This leakage in a finite-width structure, as discussed, can be represented as a summation of the modes in Fig. 4. As these modes propagate away from the launch, they get out of phase due to their different phase velocities, and construct the cone of fields characteristic of a leaky wave. The longer the transmission line, the more energy will leak away from the strip; it is not surprising, therefore, that the loss associated with S_{21} increases with increasing line length. Finally, the microstrip mode on an isotropic substrate is nonleaky. This explains why the loss in $|S_{21}|$ is seen for the Epsilam-10 and not for the Rogers substrate, the former supporting a leaky mode, and the latter not.

IV. Conclusions

The well-known phenomenon of leakage associated with printed transmission lines on substrates of infinite extent has been investigated for the case of such transmission lines on substrates of finite extent. The situation for which the substrate has infinite extent was termed Case 1, while geometries with finite transverse extent were identified as Case 2. In Case 1, the transmission-line mode becomes leaky when the mode becomes a fast wave with respect to a surface wave supported by the surrounding dielectric. The fields exist predominantly inside of a cone dictated by the angle of leakage, and are represented by a leaky-wave mode with complex propagation constant. In Case 2, all the modes have purely real or imaginary propagation constants. The presence of the box perturbs the transmission-line modes, and the presence of the strips perturbs the box modes. Thus, we described the modes in Case 2 as perturbed transmission-line and perturbed box modes. When the perturbed transmission-line mode becomes a fast wave with respect to one or more surface waves supported by the dielectric substrate, its fields are not confined to the vicinity of the strip(s); in this case, the fields away from the strip are in the form of surface waves propagating at an angle. If we excite the fields in Case 2 in the same manner in which the leaky-wave mode is excited in Case 1, the fields in Case 2 leave the launch in the form of a cone, characteristic of a leaky wave. Until the fields in Case 2 hit the sidewalls, the fields in Case 1 and Case 2 are identical. As discussed, the fields in Case 2 must be represented as a sum of spectral modes; if the box is sufficiently narrow, all the perturbed box modes are cutoff, and the complicated fields emanating from the launch can be represented by a single propagating perturbed transmission-line mode and an infinite number of cutoff modes. In many practical cases, however, the leakage angle and/or box width dictate that more than one propagating mode is required to represent the field cone leaving the launch, and eventually hitting the sidewalls. In this case, usually the perturbed transmission-line mode and one or more propagating perturbed box modes are excited. Therefore, at all distances away from the launch, more than one propagating mode is required to represent the total fields. Moreover, in the vicinity of certain discrete frequencies, the angle of the surface waves leaving the strip(s) for the perturbed transmission-line modes is equal to the angle of the surface wave associated with a particular perturbed box mode; at and around these frequencies, the perturbed transmission-line mode and perturbed box mode exhibit phenomena associated with classical coupled-mode theory.

Several of the points addressed have been detailed quantitatively with numerical computations and measurements. The calculations were performed using the finite-element method and the spectral-domain technique, and were presented in the form of dispersion curves, field plots, and power density plots. All of these results demonstrated the interaction of the perturbed transmission-line mode with perturbed box modes, and emphasized a phenomenon associated with classical coupled-mode theory. The experiments were performed on several geometries, and clearly showed that the leakage-like effect in finite width structures can lead to a severe drop in $|S_{21}|$ at frequencies for which the fields are not bound to the vicinity of the strip or slot, even if the substrate is not of infinite extent.

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